# Magnetic Field Topology Reconstruction Using Gaussian Process Regression in Machine Learning

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## INTRODUCTION

Unlike the vast majority of astrophysical plasmas, the solar wind is accessible to spacecraft, which for decades have carried in-situ instruments for directly measuring its particles and fields. Though such single-spacecraft measurements provide precise and detailed information, one such spacecraft on its own can neither disentangle spatial and temporal fluctuations nor fully reveal the plasma's 3-D structure. To address this, a few missions have flown with 4 or 5 spacecraft (e.g., Cluster, THEMIS-ARTEMIS, and MMS), and missions with even more spacecraft, for example HelioSwarm which was recently selected by NASA for phase A, have been proposed. However, none of the missions have succeeded in generating a full three dimensional image of the magnetic vector field in the solar wind, mostly because of insufficient spacecraft. A full 3-D image would provide the information related to structure and topology which are extremely important for understanding turbulence and its evolution in space plasma specially how energy is stored in and transported through the plasma. Though an active field of research, not much has been done in this from the vantage point of machine learning. In this study we present a proof of concept of magnetic field's topology reconstruction using multi-point observation in a simulation box. We show how machine learning can be employed to effectively construct 3-D structure of the plasma's 3-D magnetic filed from synthetic data.

## BACKGROUND

Machine learning is becoming increasingly relevant in data analysis techniques. In this study we implement one such technique to simulation data.

We use Gaussian Processes (GP) Regression to interpolate data between points (see section 3 for more information). GP is a probabilistic data imputation technique. The prior and posterior distributions of GP's Bayesian framework are set by input functions (kernels), whose parameters are "learned" during the regression process.

Out of many available standard kernels for GP, we report results from four selected kernels.

## KERNELS

List of kernels used in the present study:

a) Constant (C):

$$k(x_1, x_2) = \text{constant\_value} \ \forall \ x_1, x_2$$

b) Linear:

$$k(x_i, x_j) = \sigma_0^2 + x_i \cdot x_j$$

c) Rational Quadratic (RQ):

$$k(x_i, x_j) = \exp\left(-\frac{d(x_i, x_j)^2}{2l^2}\right)$$

d) Radial Basis Function (RBF):

$$k(x_i, x_j) = \left(1 + \frac{d(x_i, x_j)^2}{2\alpha l^2}\right)^{-\alpha}$$

e) Matern:

$$k(x_i, x_j) = \frac{1}{\Gamma(\nu)2^{\nu-1}}\left(\frac{\sqrt{2\nu}}{l}d(x_i, x_j)\right)^{\nu} K_\nu\left(\frac{\sqrt{2\nu}}{l}d(x_i, x_j)\right)$$

We also present results for spacecraft numbers varying from 4 to 34 at different distances from the central point for a combination of the constant and matern kernel.

## METHODOLOGY

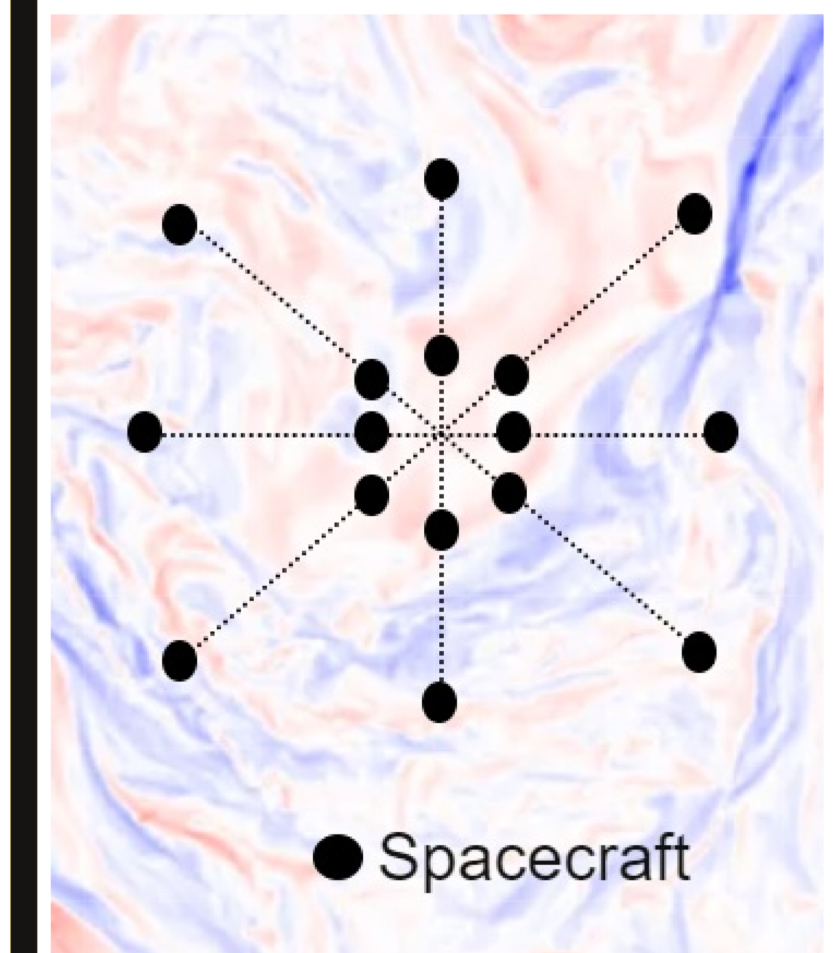


Figure1.Spacecraft configuration for 16 spacecraft

The methodology involves applying GP to a synthetically prepared in-situ time series magnetic field data obtained by flying a given configuration of a set number of spacecraft through a plasma with every spacecraft measuring magnetic field vector at each point at a certain cadence.

For the purpose of synthetically prepared data, we used a fully kinetic 3-D simulation as reported in Roytershteyn2015. The spacecraft configuration for the case of 16 spacecraft is shown in Figure 1. Each spacecraft carries a magnetometer which records magnetic field vector at 10 Hz cadence.

Separation between spacecraft is maintained such that we have measurement over at least one order of spatial magnitude. Thus, the inner most spacecraft are at a distance of 1di (ion-inertial length) where as the outer most are at spearation of 11di from the centeral hub.

In order to construct full 3D structure of the magnetic field, we apply Gaussian Process (GP) to each component of the magnetic field independently, for interpolation among all points within the given volume of space (~ 42$d_i^3$). We used the package Scikit Learn available in Python (Pedregosa2011) for implementing the gaussian processes.

## CONCLUSION

- We presented a proof of concept for reconstructing a 3-D image of synthetically prepared in-situ measurement of magnetic field in space plasma to study its structure. The data was prepared for a theoretical constellation of spacecraft flying through solar wind. Gaussian Processes Regression was used for interpolation in order to get the complete picture.
- Based on preliminary results, we conclude that around 24 spacecraft would be required to reproduce structure with any reliability. And among the kernels used, a combination of the constant and Matern kernel seem to provide best image reconstruction.
- Ongoing studies are exploring the impact of the spacing and arrangement of the spacecraft on the quality of the magnetic reconstruction.

## RESULTS AND DISCUSSIONS

Figure 2 shows original, panel (a) and reconstructed x-component of magnetic field for one specific xy-plane (panels b to f). Panel (b) which shows the reconstructed image using constant and Matern kernel shows the most similar structure. Though, both linear and RBF kernels produces decent reconstruction, a combination of two produces very erroneous image.

Based on these results, we focused on combination of constant and Matern kernel. Figure 3 shows a similar reconstruction for various numbers and arrangements of spacecraft.

We observe that small number of spacecraft (4-8) completely fails to provide any meaningful information about the structure of the magnetic field. It is only when we use 16 or more spacecraft that we see a resembelance of structure in reconstructed image.

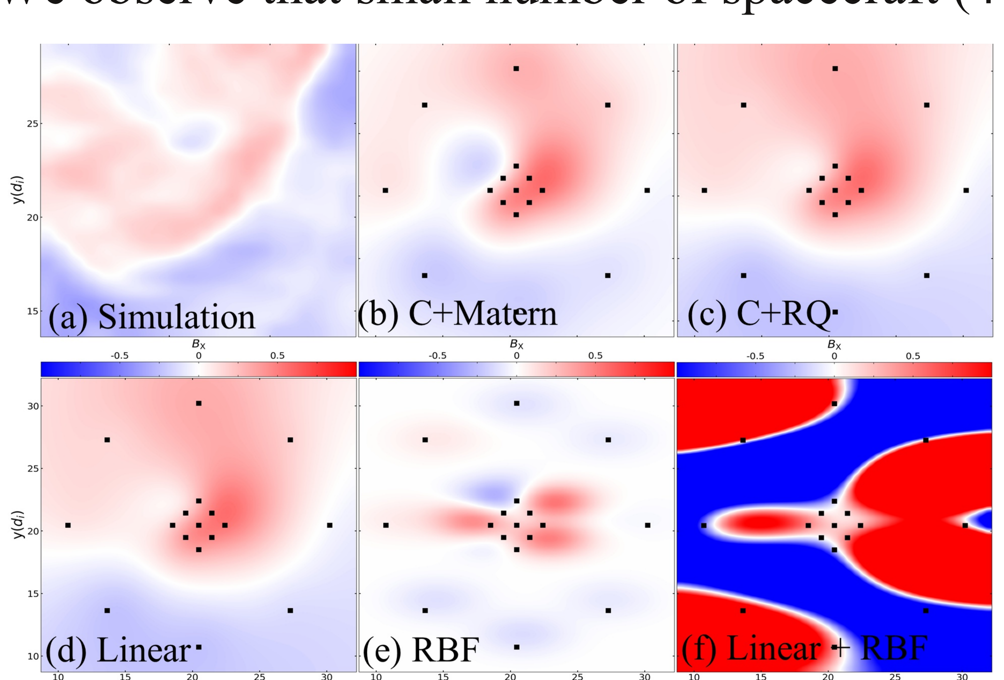


Figure 2. Original and reconstructed x-component of magnetic field for different kernels.

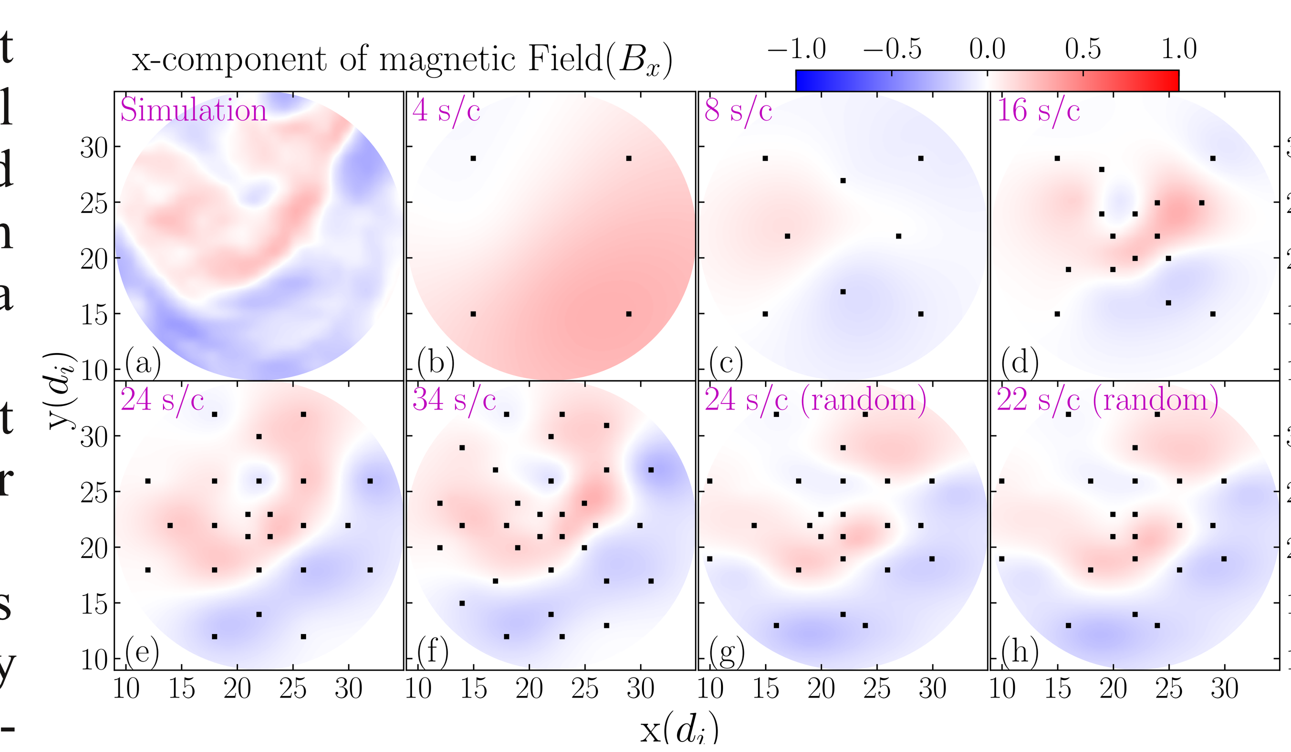


Figure 3. Original and reconstructed x-component of magnetic field for different number and configuration of spacecraft. Each black dot is position of spacecraft.

Increasing the number of spacecraft from 24 to 34 slightly improves the quality of reconstructed image.

Last two panels ( g and h) show the images for a configuration where 24 and 22 spacecraft are distributed randomly. The quality of image remains high.

## REFERENCES

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