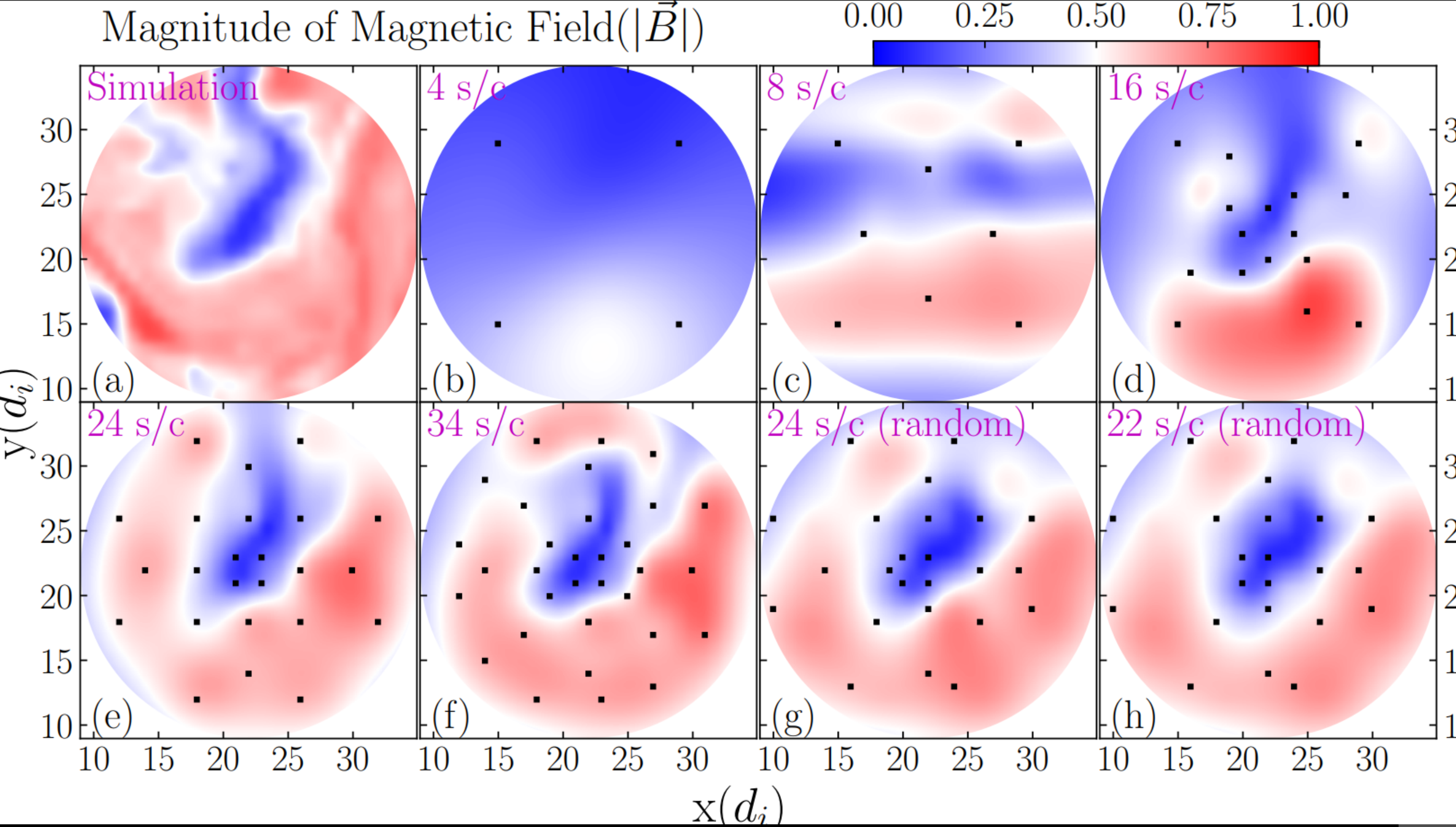
Gaussian Process Regression for Magnetic Field Reconstruction



https://slides.com/qudsi/magnetore

Some relevant scales:

$$d_i(1\mathrm{AU}) \sim 100 \text{ km}$$

$$V_{sw} \sim 500 \text{ km/sec}$$

$$X_{sim}(\text{boxsize}) \sim 40 d_i$$

$$\sim 4 \times 10^3 \text{km}$$

$$d_{spc} \sim [1, 11]\, d_i$$

$$\sim [10^2, 10^3] \text{ km}$$

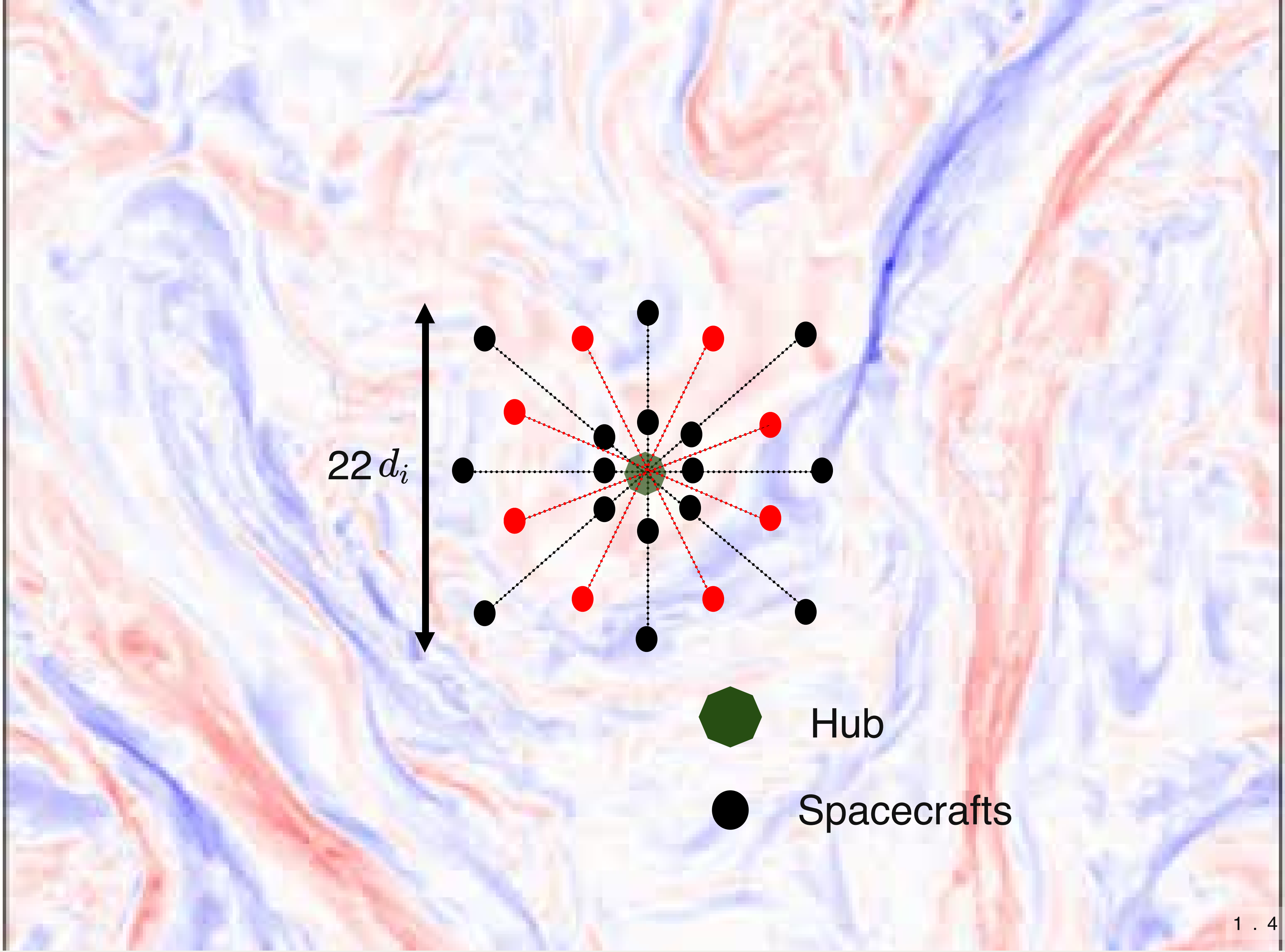
$$f[min, max] \sim V_{sw}/(2 \times d_{spc,max,min})$$

$$\sim [0.25, 2.5] \text{ Hz}$$

$$\nu(\text{sampling rate}) \sim 40\text{Hz}$$

$$(\text{just because of box size and stuff})$$

$$\text{time to move across the simulation box} \sim 10 \text{ sec}$$



$22\,d_i$
Hub
Spacecrafts

Gaussian Process Regression Interpolation

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Gaussian Processes

$$f(\mathbf{x}) \sim \mathcal{GP}\left(m(\mathbf{x}), k(\mathbf{x}, \mathbf{x}')\right)$$

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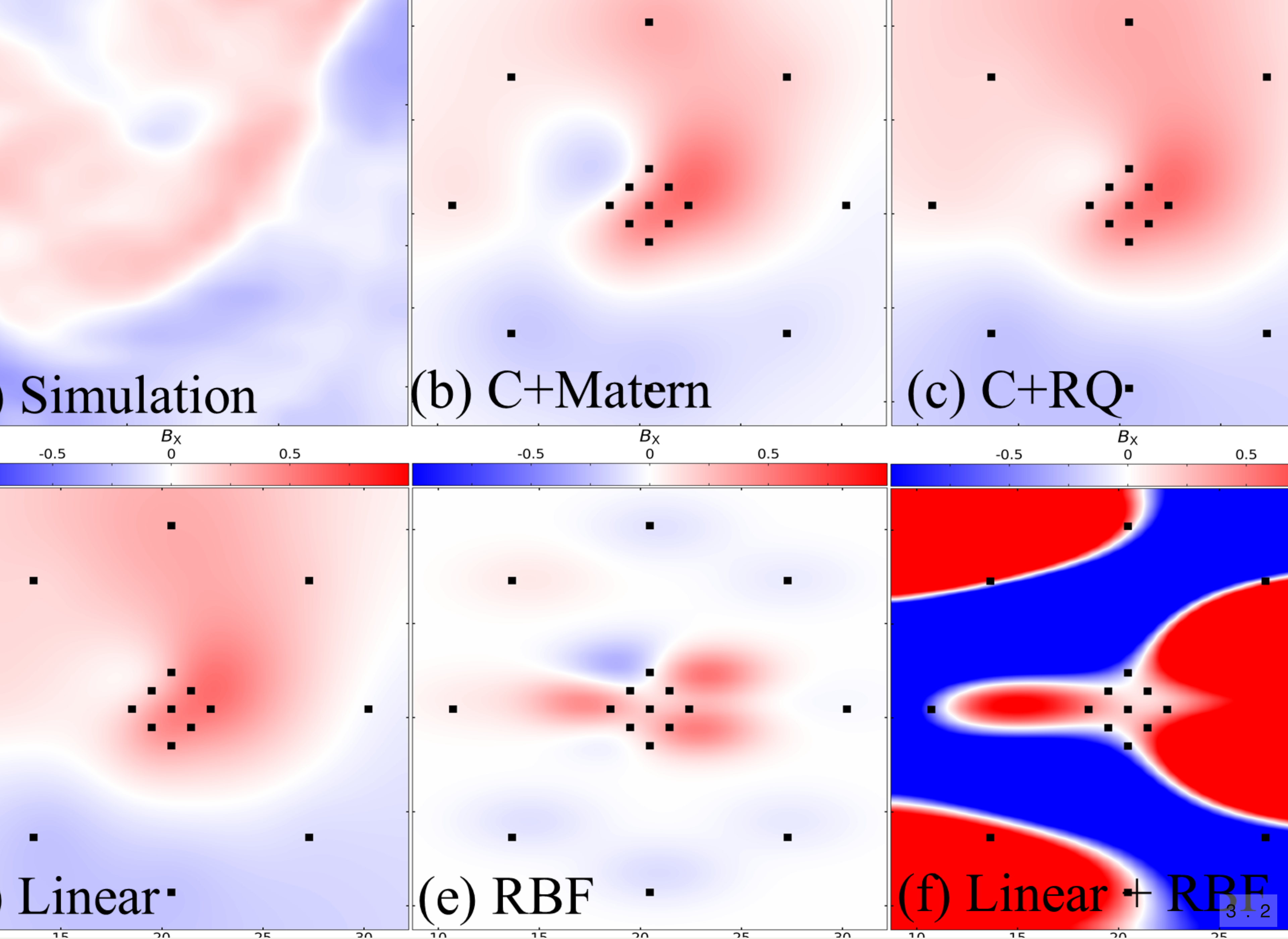
$$k(x_i, x_j) = \frac{1}{\Gamma(\nu)2^{\nu-1}}\left(\frac{\sqrt{2\nu}}{l}d(x_i, x_j)\right)^{\nu}$$ ← Matern

$$K_\nu\left(\frac{\sqrt{2\nu}}{l}d(x_i, x_j)\right)$$

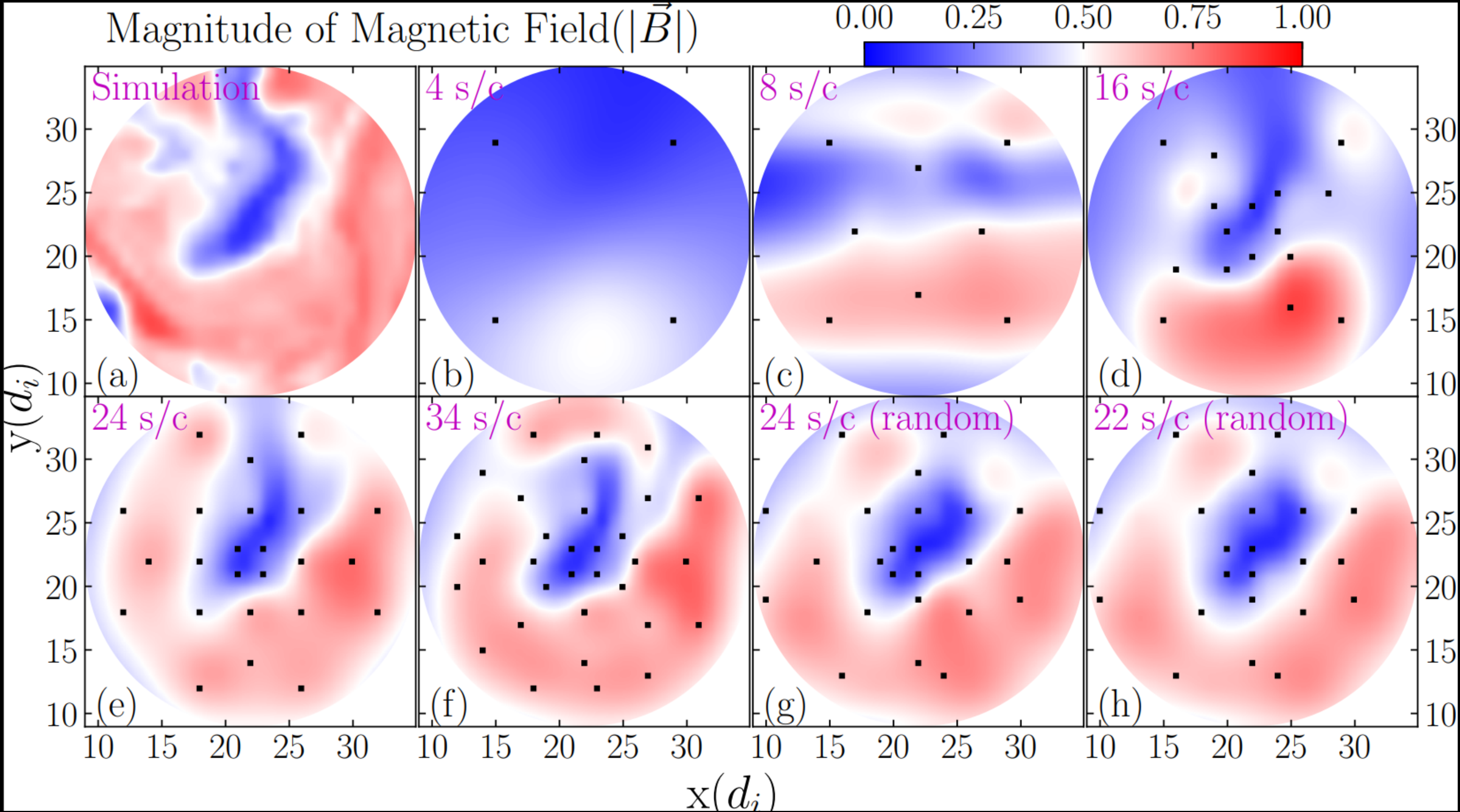
https://scikit-learn.org/stable/modules/gaussian_process.html

```python

# Define the plane an ddirection of the motion of spacecrafts
pln = 'xy'
drn = 'z'
ck_len = 5
mat_len_scl = [2,2,6]
mat_nu = 5/2
sigma_0 = 0
#Define the kernel you want to use and the associated required parameters
#kernel = CK(1, (1e-2, 1e2)) * RBF([2,2,2], (1e-2, 1e2))
#kernel = CK(ck_len, (1e-2, 1e2)) * Matern(length_scale=mat_len_scl, nu=mat_nu) #The relevant one
#kernel = CK(ck_len, (1e-2, 1e2)) + CK(ck_len, (1e-2, 1e2)) *\
#            Matern(length_scale=mat_len_scl, nu=mat_nu)
#kernel = CK(5, (1e-2, 1e2)) * Matern(length_scale=2, nu=3/2)
#kernel = CK(1.0, (1e-2, 1e2)) * RQ(length_scale=1, length_scale_bounds=(1e-2, 1e2))
#kernel = RBF([2,2,2], (1e-3, 1e3))
#kernel = DP( sigma_0=1.0, sigma_0_bounds=(1e-02, 100.0)) * RBF([2,2,2], (1e-3, 1e3))
kernel = DP(sigma_0=sigma_0, sigma_0_bounds=(1e-02, 100.0)) + CK(ck_len, (1e-2, 1e2))\
            * Matern(length_scale=mat_len_scl, nu=mat_nu)

# Define the gaussian processor regressor, keep the number of optimizers low, preferably within 100,
# thoug it is fine if you want to go for a higher number. However, if you do go for a higher number,
# please do so with ample caution

print('Running Gaussian processes')
n_restarts_optimizer = 18
gp = GaussianProcessRegressor(kernel=kernel, n_restarts_optimizer=n_restarts_optimizer)

#Get the model based on the defined kernel and test dataset
gp.fit(X, y)

print('Fit model created')

y_pred, MSE = gp.predict(x1x2x3, return_std=True)
Zp = np.reshape(y_pred,(indx_max - indx_min + 1, indy_max - indy_min + 1,
            indz_max - indz_min + 1))
MSE = np.reshape(MSE,(indx_max - indx_min + 1, indy_max - indy_min + 1,
            indz_max - indz_min + 1))
print('Done running Gaussian Processes successfully \n')
print('Saving GP data to a custom file name')
```

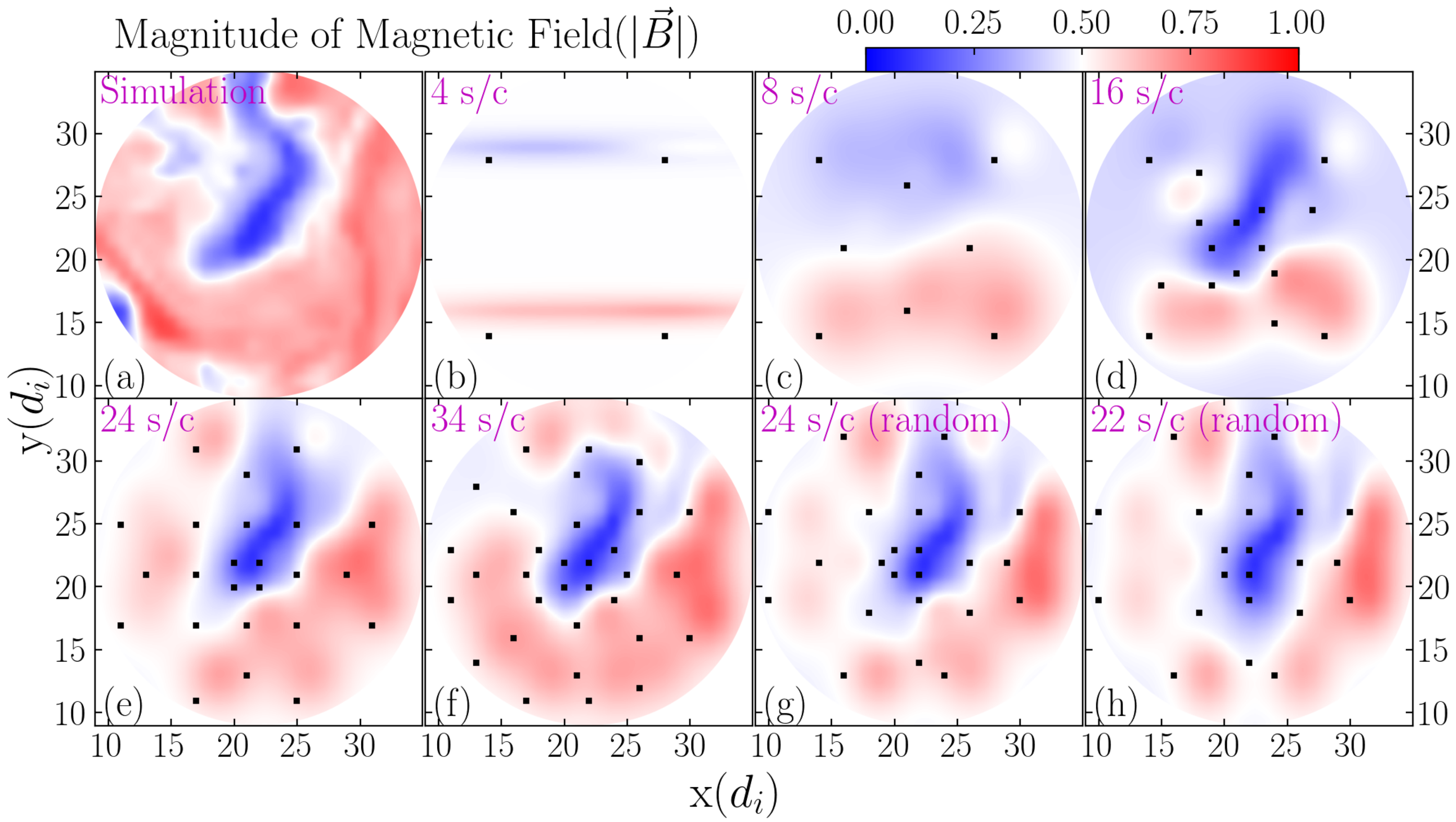
Results



) Simulation
(b) C+Matern
(c) C+RQ
B_X
-0.5
0
0.5
B_X
-0.5
0
0.5
B_X
-0.5
0
0.5
) Linear
(e) RBF
(f) Linear + RBF
3 . 2



One component at a time



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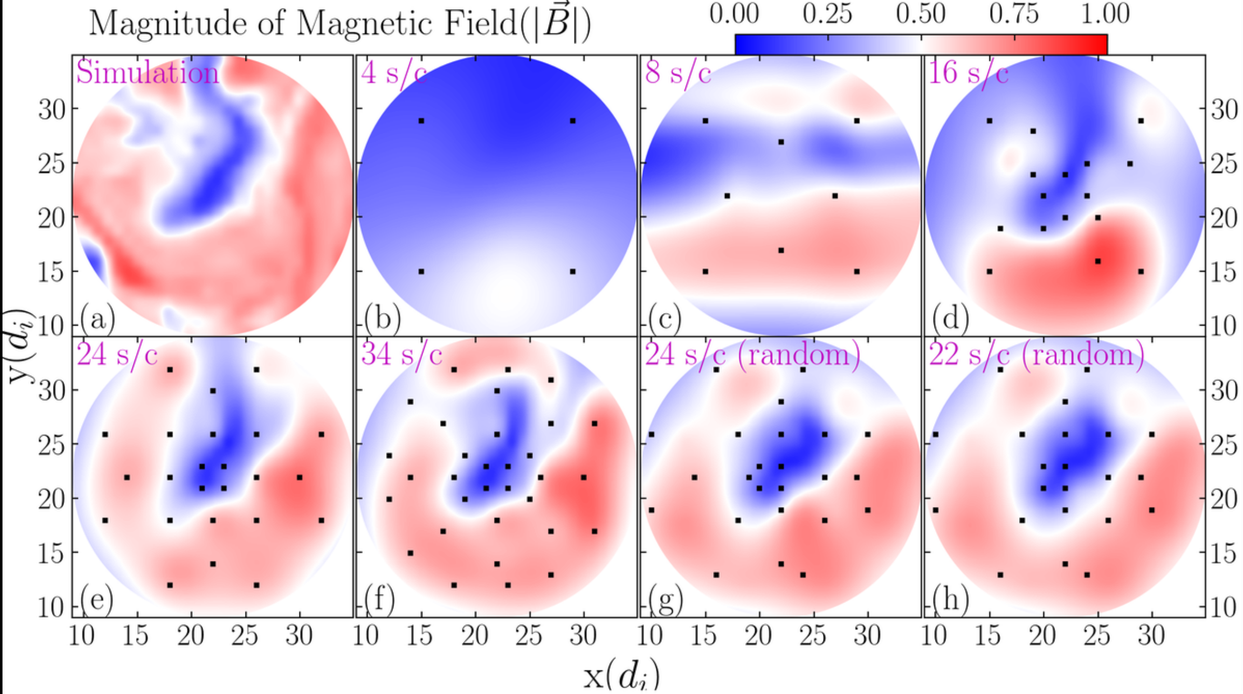
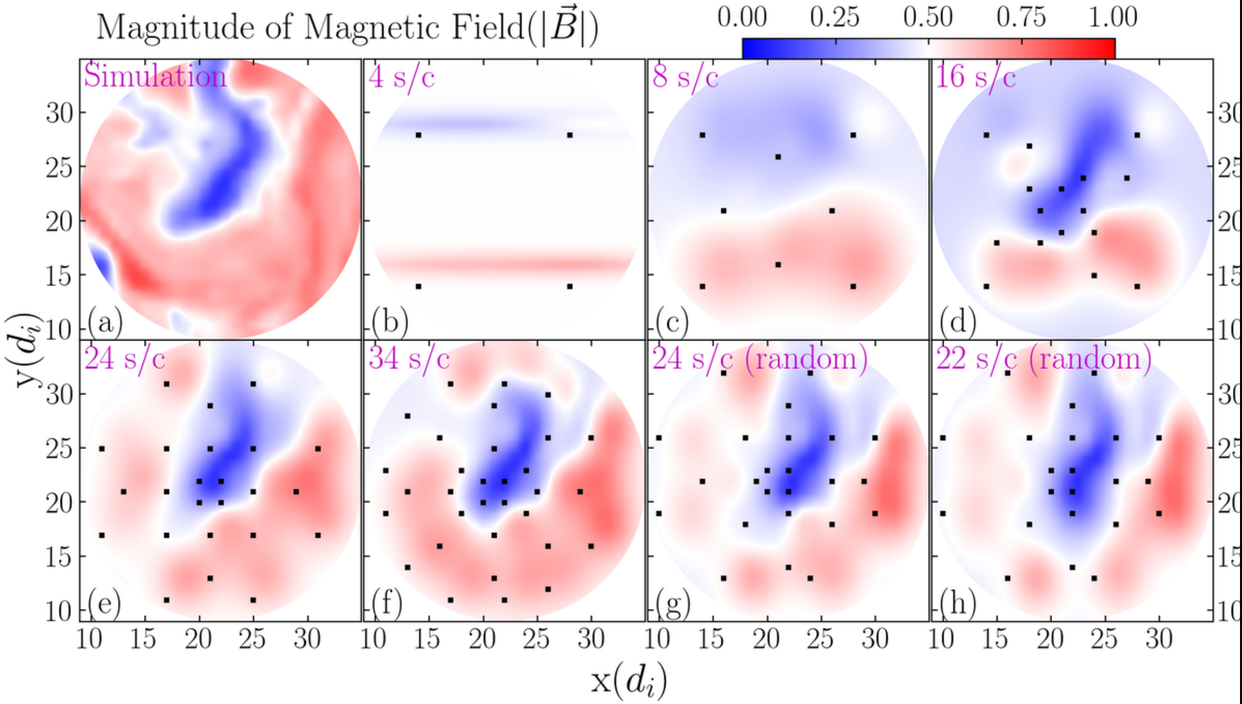
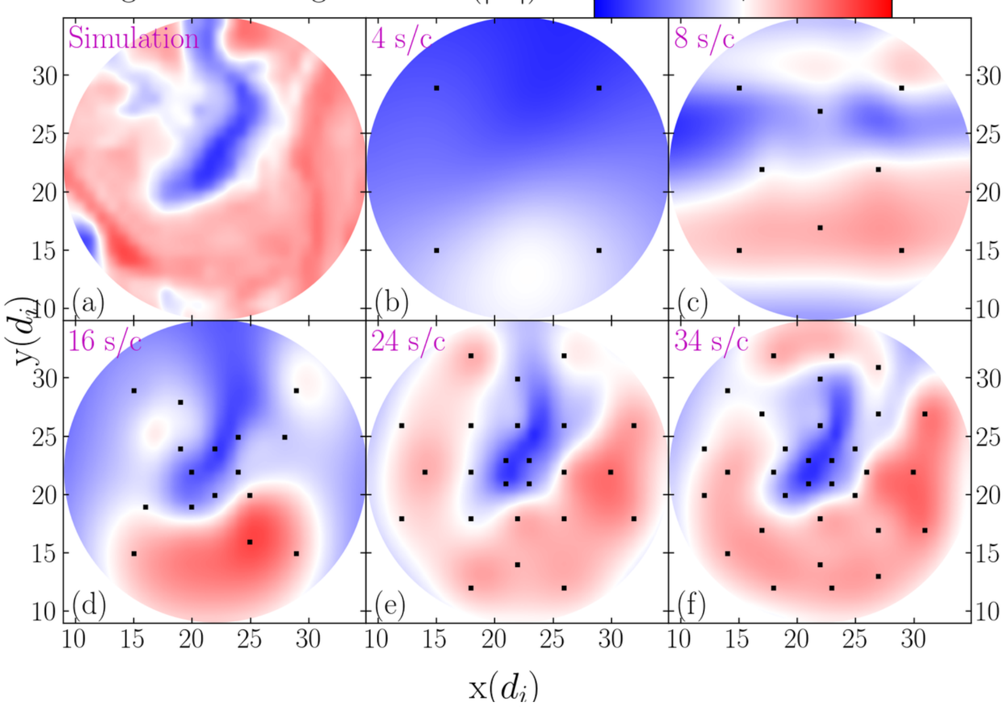
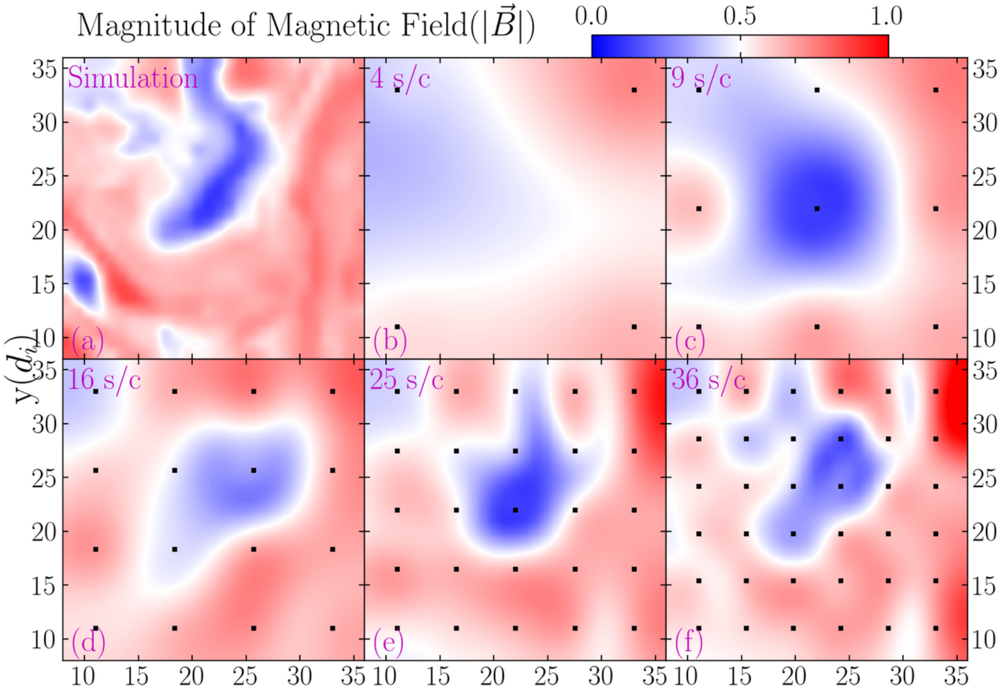


Single component input

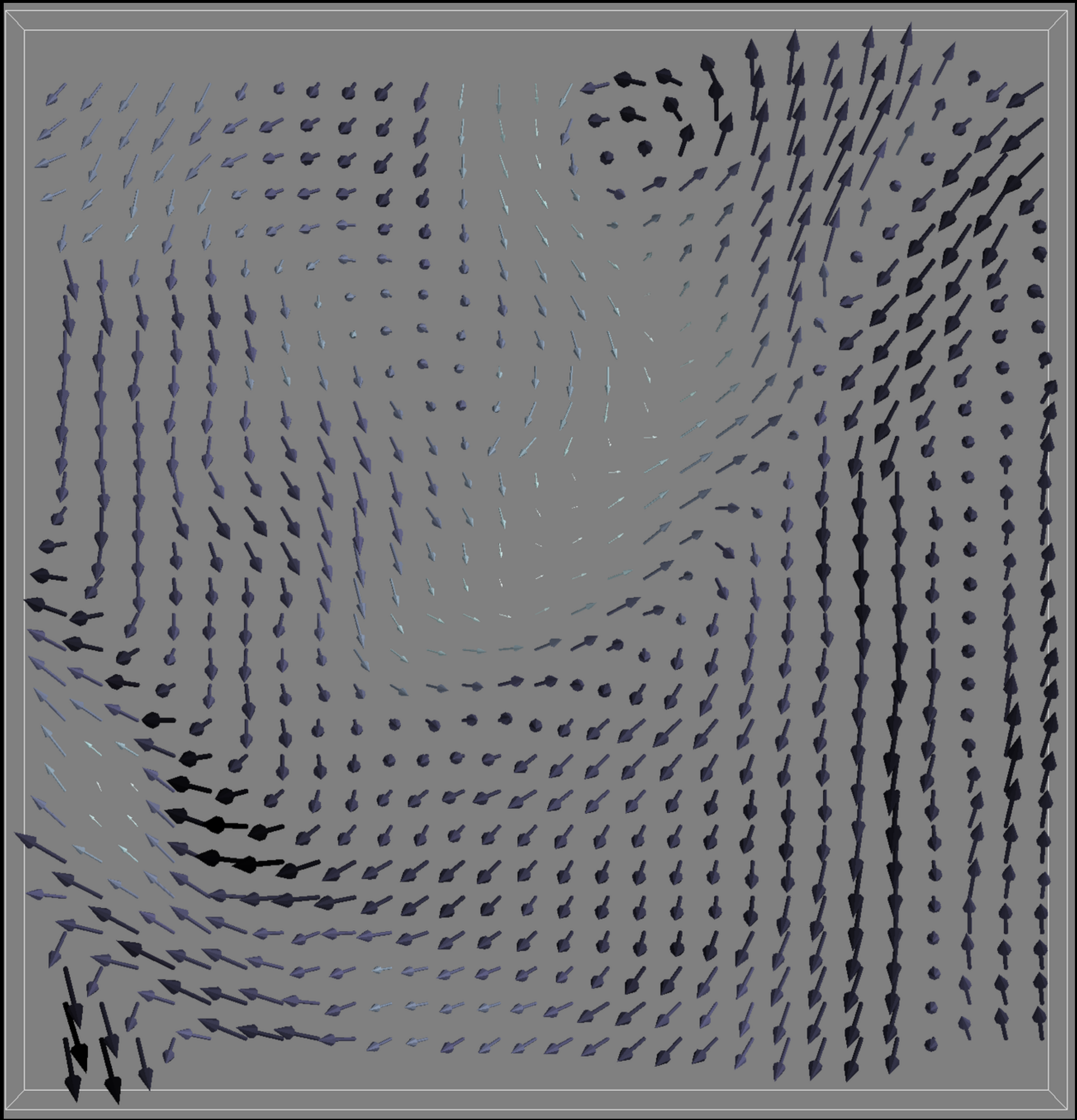


Vector input

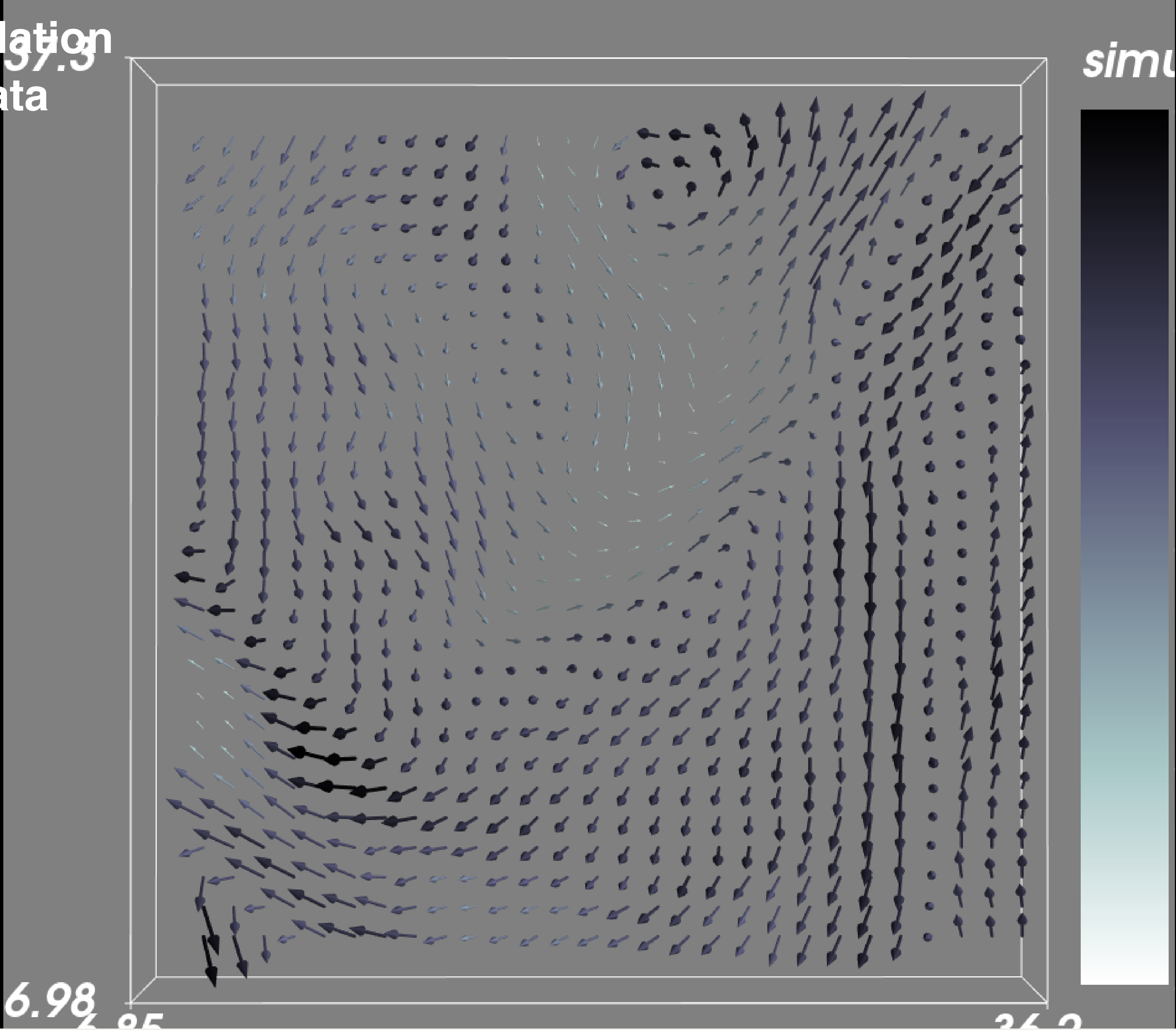

Simulation
4 s/c
8 s/c
16 s/c
24 s/c
34 s/c
(a)
(b)
(c)
(d)
(e)
(f)
$y(d_i)$
$x(d_i)$


Magnitude of Magnetic Field($|\vec{B}|$)
0.0
0.5
1.0
Simulation
4 s/c
9 s/c
16 s/c
25 s/c
36 s/c
(a)
(b)
(c)
(d)
(e)
(f)
$y(d_i)$

simulation data

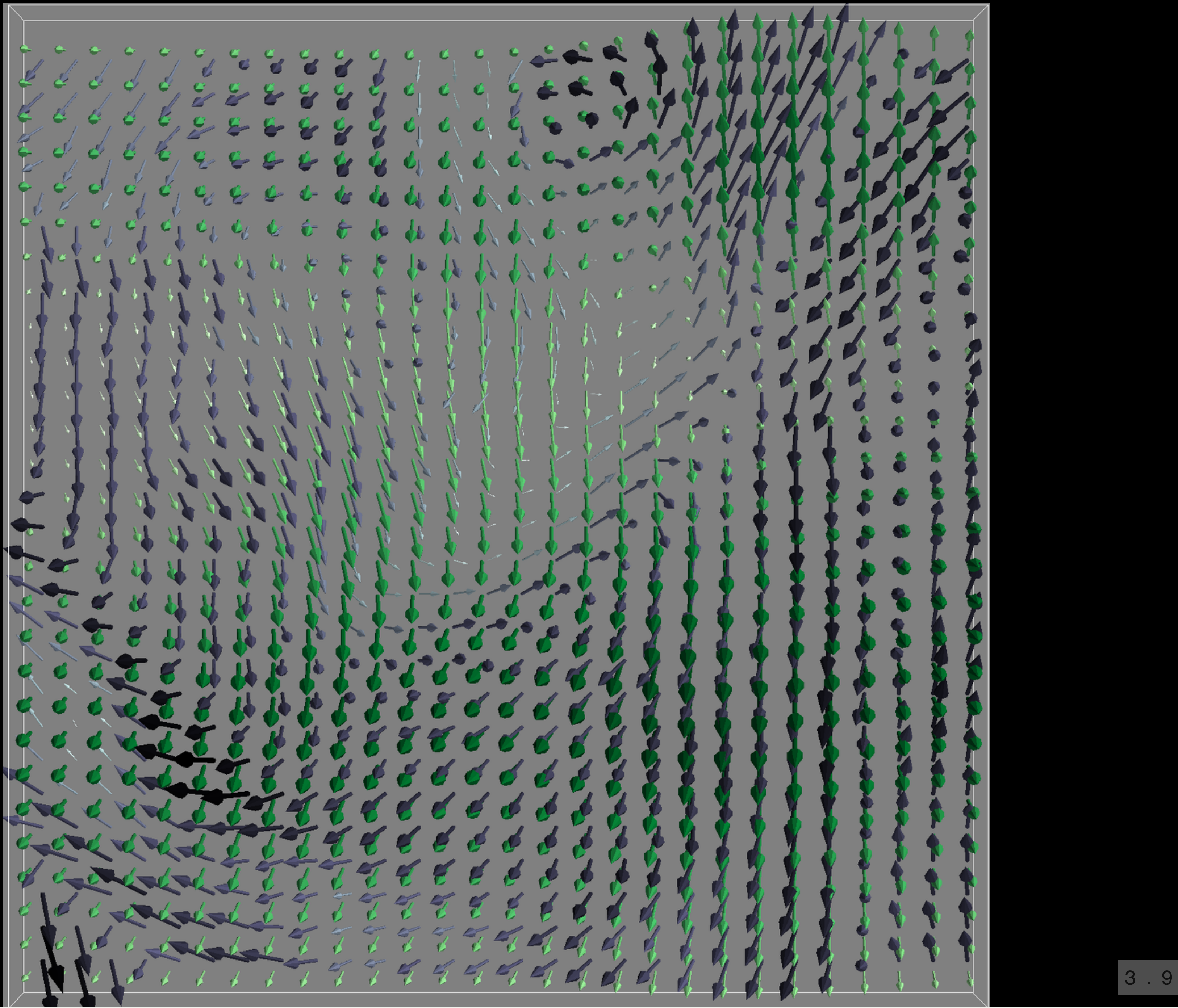


simulation data

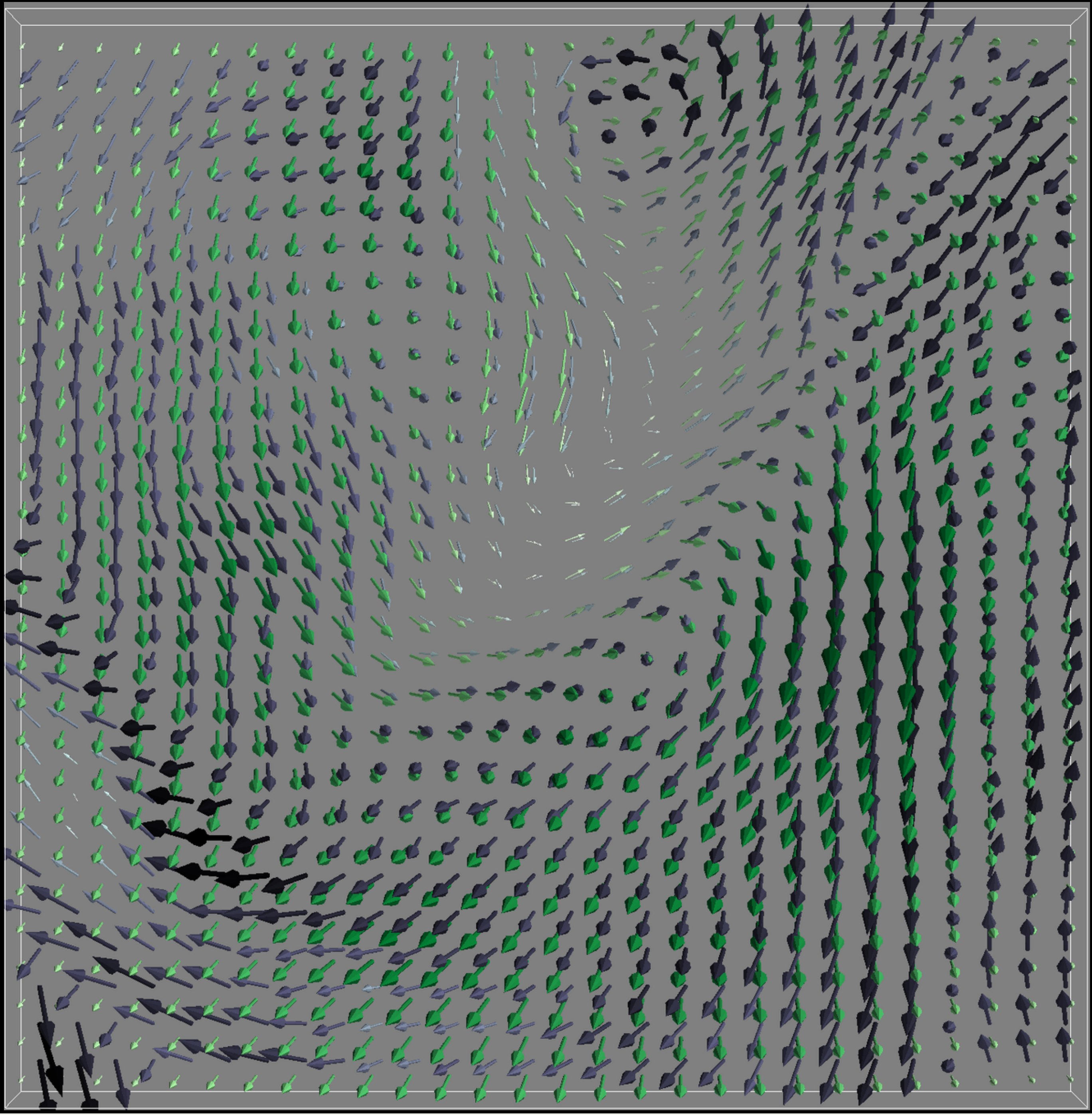


3 . 8

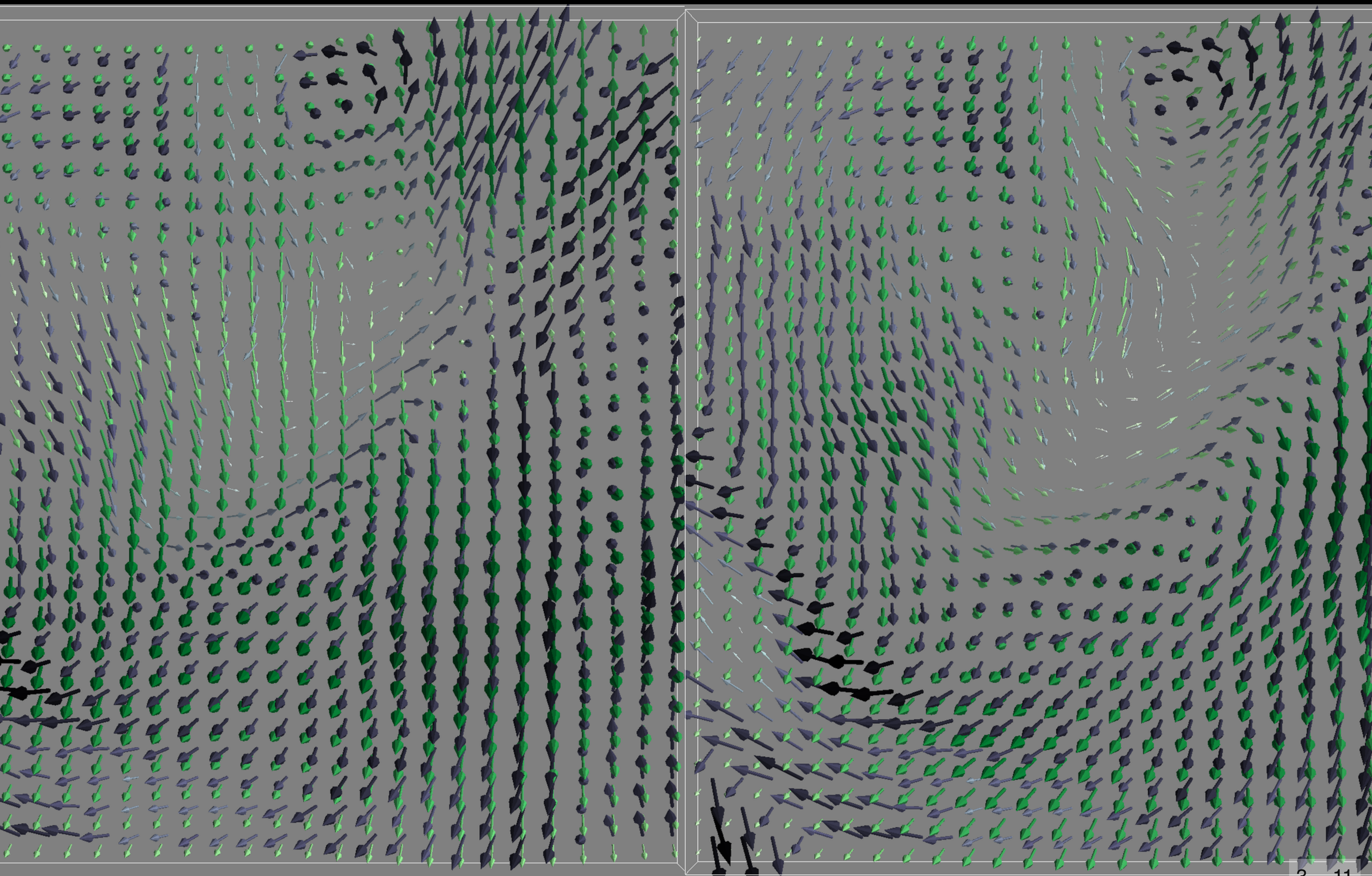
8 spc

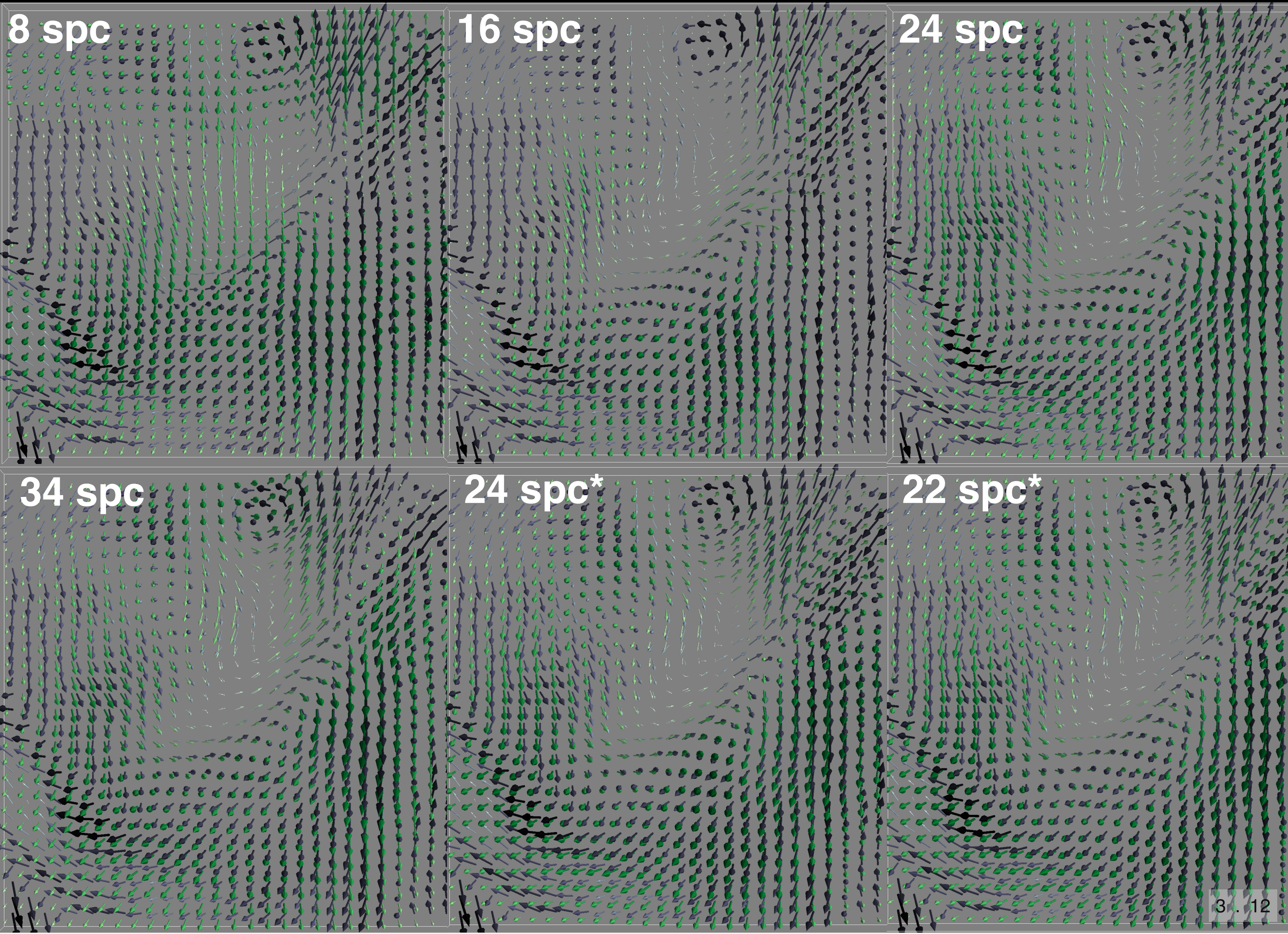


24 spc



8 spc **24 spc**




8 spc
16 spc
24 spc
34 spc
24 spc*
22 spc*
3 . 12

Moving Forward

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- Explore the impact of relative spacecraft separation and arrangement
- Incorporate the zero divergence condition on magnetic field
- Explore alternative algorithms/methods
  - Deep Gaussian Processes
  - Neural Network as GP

Thank You!